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Predators in the market:
Implications of market interaction on optimal resource management

Stein Ivar Steinshamn

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1 INTRODUCTION

Analysis of multi-species and ecosystem models has been common in the bioeconomic literature at least since the 70s, whether it has been for the purpose of studying open access, maximum yield or economic rent; see e.g. Anderson (1975), Silvert and Smith (1977) and May et al. (1979). More recent contributions include Kasperski (2015) and Wang and Ewald (2010). In these articles, however, the interaction between species has always been biological, ecological and sometimes technical, but rarely in the market. Most articles that take market-interactions into account, are empirical studies, and many, if not most of them, seem to deal with interaction between aquaculture and wild caught fish (Anderson, 1985; Ye and Beddington, 1996).

Analysis of substitutes and complements in demand is fundamental in economics and well known from basic textbooks as well as numerous empirical studies, e.g. Meng (2014) and Garcia and Raya (2011) to mention a couple of recent ones. This phenomenon also applies to natural resources such as fish products (Vignes and Etienne, 2011). However, there are only a few studies that systematically investigate implications of cross-price effects on optimal management of renewable resources from a conceptual and theoretical angle, probably because such models have a tendency to become very messy. There are, however, some recent exceptions to this rule, and Quaas et al. (2013) is one such. Their results are based on the assumption that there are two separate stocks, which by coincidence are symmetric in the sense that they have identical functional forms and identical parameters in the growth function. Using this assumption they find that the problem simplifies quite a bit. Quaas and Requate (2013) study the effects of preferences for diversity in a model with an arbitrary number of fish species. An older example is Ruseski (1999) who uses a two-stage, two-period model to analyze the behavior of two agents, one regulated and one unregulated, who harvest identical products from two separate stocks. He finds that trade in the presence of market power and divergent management regimes may produce unexpected results.

In this article a continuous time two-species bioeconomic model is applied to investigate the effects of economic (market) interaction between species on optimal management from a sole-owner perspective. That is, the owner, or manager, of both species is one and the same who maximizes the combined revenue from the two stocks. This may seem far-fetched if the term sole-owner is taken literally. But here the more common interpretation of the term sole-owner is used, namely that it represents the managing authority of a nation who behave as a sole-owner on behalf of its inhabitants in order to maximize the aggregated resource rent. There may, for example, exist two stocks in different parts of the country's EEZ, but with certain similar characteristics making them substitutes in the market. Such characteristics may, for example, be that both species are "white fish" or that they are used for fish-meal or fish-oil production. This sole-owner exploits a certain degree of direct and indirect market power, and the demand functions are assumed to be stationary over time.

The biological model is a surplus growth model, but the only interaction between the species is in the market where the quantity of each species may affect the price of the other. The aim of this study is to investigate implications of market interaction upon the optimal steady state and on the paths leading to the steady state. Revenue and costs for each species are separable, but the harvest of one species enters the inverse demand function for the other. There are several possibilities, for example that one species affects the price of the other, but not vice versa. The most realistic assumption is probably that they are true substitutes such that both species affect the other species' price. Somewhat facetiously, we can say that they predate on each others price. No technical or biological interactions are considered.

The analysis is divided in two parts, first a steady state analysis and then a dynamic analysis. Each of these parts are again divided in two, based on whether the net revenue function depends on the state variable(s) or not. This is because the state variables turn out to play an important role for the results. In the steady state analysis, the results are derived analytically from the mathematical model. In the dynamic analysis, on the other hand, numerical methods are resorted to as it is beyond realistic expectations to hope for closed-form solutions of a highly non-linear system of four differential equations.

2 THE GENERIC MODEL

The model is a continuous-time, bioeconomic model of the surplus-growth type, with two species, x and y, but with no biological interaction. The two species are assumed to be substitutes in the market implying that the cross-price elasticities are negative. In other words, the price of one species may depend on both own supply and the supply of the other species, and therefore there exist certain degrees of market power that are exploited. The generic inverse demand functions look as follows:

$$p_x = p_x(h_x, h_y)$$

$$p_y = p_y(h_x, h_y)$$

where p_i is price of species i and h_i is harvest of species i. Technically it is assumed that $\partial p_i / \partial h_j < 0$ for $i, j \in (x, y)$. The net revenue function, in its most generic form, is then defined as

$$R(x, y, h_x, h_y) = p_x(h_x, h_y)h_x + p_y(h_x, h_y)h_y - \kappa_x(x, h_x) - \kappa_y(y, h_y) \tag{1}$$

where x and y denote the size of the respective stocks, and κ_x and κ_y are cost functions. The separability of the cost functions rule out technical interactions.

The net revenue function is the objective function to be maximized with respect to h_x and h_y as control variables, whereas x and y are the state variables. The state and control variables are all functions of time, t. In addition there are two separate biological surplus growth functions, one for each species: $f(x)$ and $g(y)$. The infinite horizon dynamic optimization problem resulting from this leads to the following discounted Hamiltonian:

$$H = e^{-\delta t} R(x, y, h_x, h_y) + \lambda[f(x) - h_x] + \gamma[g(y) - h_y]$$

where λ and γ are costate variables, also functions of t, and δ is the discount rate. The first-order conditions for this general case are given by[1]

$$\partial H/\partial h_x = \partial H/\partial h_y = 0 \tag{2}$$

and

$$d\lambda/dt = -\partial H/\partial x, \;\; d\gamma/dt = -\partial H/\partial y$$

together with the dynamic constraints

$$dx/dt = f(x) - h_x \tag{3}$$

$$dy/dt = g(y) - h_y \tag{4}$$

and initial conditions $x(0) = x_0$ and $y(0) = y_0$. Now let R with subscripts represent the first and second partial derivatives with respect to its respective arguments as defined in Eq. (1). For example $R_1 \equiv \partial R/\partial x$ and $R_{12} \equiv \partial^2 R/\partial x \partial y$.

[1] It is assumed that H is continuous, strictly concave and twice differentiable in the control variables h_x and h_y. Concavity in H is fulfilled when demand is downward sloping and the cost functions are convex.

From the general definition of R it is seen that

$$R_{12} = R_{14} = R_{21} = R_{23} = R_{32} = R_{41} = 0.$$

The first-order conditions solved with respect to the discount rate yield the following two criteria:

$$\delta = f^{'}(x) + \frac{R_{31}}{R_3}\frac{dx}{dt} + \frac{R_{33}}{R_3}\frac{dh_x}{dt} + \frac{R_{34}}{R_3}\frac{dh_y}{dt} + \frac{R_1}{R_3} \tag{5}$$

$$\delta = g^{'}(y) + \frac{R_{42}}{R_4}\frac{dy}{dt} + \frac{R_{43}}{R_4}\frac{dh_x}{dt} + \frac{R_{44}}{R_4}\frac{dh_y}{dt} + \frac{R_2}{R_4}. \tag{6}$$

The two equations in (2) can be used to find explicit solutions for the costate variables and hence their time derivatives. Taking the first-order conditions and solving for $dx/dt, dy/dt, dh_x/dt$ and dh_y/dt by eliminating the costate variables and their time derivatives, yields the following system of non-linear first-order differential equations:

$$dh_x/dt = \frac{R_{34}A - R_{44}B}{C}$$

$$dh_y/dt = \frac{-R_{33}A + R_{43}B}{C}$$

where[2]

$$A = R_4(g^{'} - \delta) + R_{42}(g - h_y) + R_2$$

$$B = R_3(f^{'} - \delta) + R_{31}(f - h_x) + R_1$$

$$C = R_{33}R_{44} - R_{34}R_{43}.$$

Together with the dynamic constraints (3) and (4), using $R_{32} = R_{41} = 0$, this constitutes a system of four differential equations. In the following we assume decreasing marginal return on harvest, that is $R_{33} < 0$ and $R_{44} < 0$. If, in

[2] It is worth noticing that A depends on y and h_y whereas B depends on x and h_x.

addition, we assume $C > 0$, then R will be partially convex in the control variables. This is fulfilled if the direct price effect is stronger than the cross-price effect, which seems to be a reasonable assumption. Restricting the analysis to the closed intervals $0 < h_i < MSY_i$, and x and y to be between zero and the natural carrying capacity, will guarantee the existence of both a maximum and minimum in the control variables on this interval. In the following, focus will be on interior solutions when they exist.

Finding closed form solutions for the time paths $x(t)$, $y(t)$, $h_x(t)$ and $h_y(t)$ for this system is far too optimistic, even in the simplest case. In stead, in the section Dynamic Analysis the system will be solved numerically. But first we will look at steady states.

3 STEADY STATE ANALYSIS

In this section the properties of steady states are analyzed, and it is all based on the fairly general formulation of the net revenue function found in (1). By setting all time derivatives equal to zero, it is seen that the criteria (5) and (6) simplify to the following in steady state:

$$\delta = f^{'}(x) + \frac{R_1}{R_3} \tag{7}$$

$$\delta = g^{'}(y) + \frac{R_2}{R_4} \tag{8}$$

These two equations together with $h_x = f(x)$ and $h_y = g(y)$ yield four equations to be solved for x, y, h_x and h_y. The following analysis is divided in two parts, namely when the net revenue (in practice costs) depends on the state variables x and y, and when it does not. These two cases can be thought of as representing purse seine technology and trawl technology, respectively. With purse seine technology there is usually little or no relationship between total stock size and costs of harvest whereas for trawl technology it is believed to be a strong relationship between stock size and costs.

3.1 State-independent net revenue

When costs do not depend on the stock size, all costs can technically be integrated in the demand function by defining the price as a price net of costs. Then an interesting conclusion can be made directly from observing the two simple expressions (7) and (8). This is stated in the following proposition:

Proposition 1

The optimal steady state stock and harvest levels will only depend on own- and cross-price parameters if the net revenue function does not depend on stock levels of the two species.

Proof: If the stock levels are not explicitly included in the revenue function, or the derivatives is zero, the last terms in (7) and (8) will disappear as $R_1 = R_2 = 0$. Then these two equations will be two independent equations in x and y, and the steady state will only depend on biological parameters and the discount rate ■

The implication of Proposition 1 is that the Golden Rule is exactly the same in a two-species model with market interactions between the species as it would be with two single-species models without any interaction whatsoever, namely that the marginal biological productivity of both stocks should equal the alternative rate of return represented by the discount rate. As such, this is a generalization of the same result from single-species models. Mathematically it may look simple, but thinking about it, this is a fairly strong and far from obvious observation. Let us put it this way: If the quantity of herring in the market affects the price of mackerel and vice versa, this will not affect the optimal standing stock levels of mackerel or herring, nor their corresponding harvest levels, as the technology in these two fisheries are purse seine technology. If, on the other hand, the quantity of haddock in the market affects the price of cod and vice versa, this will affect the optimal standing stock and corresponding harvest levels as the fisheries in question are characterized by bottom trawl technology where the size of the stock has strong impact on the cost of harvesting.

Corollary 1

The steady state stock for one of the species, for example x, may depend on the harvest of the other, y, even if the opposite is not true. This happens when $R_1 \neq 0$ but $R_2 = 0$ or vice versa.

Proof: This follows directly from (7) and (8)■

But even in the case where the optimal steady state is not affected by the cross-price parameters, the paths towards the steady will typically be affected irrespective of technology, as we shall see later. In practice, the way stock levels affect net revenue is through the cost functions. More specifically, therefore, if the cost functions are stock independent, optimal steady states will be characterized by the condition that marginal biological growth should equal the discount rate. In the special case that the discount rate is zero, the optimal steady states will correspond to the maximum sustainable yield levels. One practical implication of this is that cross-price effects do not make any difference with respect to steady states in schooling (purse seine) fisheries whereas they may make a difference in demersal (trawl) fisheries.

3.2 State-dependent net revenue

With state-dependent net revenue the last terms in Eqs. (7) and (8), come into play as R_1 and R_2 are no longer zero. The cross-price parameters enter the equations through the denominator of the last term, namely R_3 and R_4. From

(1) it is seen that these are given as

$$R_3 = \frac{\partial p_x}{\partial h_x} h_x + p_x + \frac{\partial p_y}{\partial h_x} h_y - \frac{\partial \kappa_x}{\partial h_x} \tag{9}$$

$$R_4 = \frac{\partial p_x}{\partial h_y} h_x + p_y + \frac{\partial p_y}{\partial h_y} h_y - \frac{\partial \kappa_y}{\partial h_y}. \tag{10}$$

It is the cross-price parameters that are of interest here, and these are $\frac{\partial p_y}{\partial h_x} < 0$ in (9) and $\frac{\partial p_x}{\partial h_y} < 0$ in (10). Let us first concentrate on R_3, as the analysis of R_4 is equivalent. As the two species are supposed to be substitutes, the cross-price elasticities are negative implying that R_3 is smaller when the cross-price effect is taken into account than if the species are economically independent, that is $\frac{\partial p_y}{\partial h_x} = 0$. This will unambiguously lead to a higher steady state stock and a more conservative harvest policy. This can be stated in the following proposition:

Proposition 2

With a strictly concave growth function, positive marginal revenue of harvest and net revenue that depends positively on the stock level for one of the stocks, then if the harvest of this species reduces the price of the other species, this implies a higher steady state stock in optimum for the stock in question.

Proof: Assume that the only cross-price effect present is from h_x to p_y. Then it is seen from (9) that having such a cross-price effect compared to not having it, will reduce R_3 through the term $\frac{\partial p_y}{\partial h_x} < 0$. As $R_1 > 0$, reducing R_3 will make the fraction R_1/R_3 larger. From (7) it is seen that making R_1/R_3 larger has to be compensated by a smaller $f'(x)$ for a given δ. R_3 and $f'(x)$ therefore goes in then same direction. As f is assumed to be concave, smaller $f'(x)$ implies going to the right (higher stock). Exactly the same reasoning applies to (10).■

Proposition 2 says that if the harvest of x affects the price of y negatively, then this will imply a higher optimal standing stock of x, and vice versa. The intuition is that the downward pressure on revenue from the other species can be regarded as an addition to the marginal cost for the sole owner, and therefore we have the well-known phenomenon that higher costs have a conservative effect. The interesting thing is that this only comes into effect when net revenue also depends on the stock. In practice, it implies that for demersal fisheries, where we expect high stock dependence of costs, cross-price relationships play a conservative role whereas for schooling fish stock (typical pelagic fisheries) cross-price relationships have little or no effect. This is an important result as it adds to the well-known fact that schooling species are already most vulnerable and exposed to extinction and collapse due to the technology in the fishery, which usually is purse seine. Demersal species caught by trawl, on the other hand, is to large extent naturally protected by their behavior (uniform distribution in the ocean) which makes it extremely costly to harvest on very small stocks even under open access regimes.

Even though (7) and (8) are easy to relate to conceptually, closed-form solutions for the steady state levels are almost impossible to find except for the simplest specifications of demand and cost functions, and even in these cases the expressions tend to become too long and messy to be of any practical value.

3.2.1 Linear demand

In the case of linear demand functions, that is when

$$p_x(h_x, h_y) = a_x - b_x h_x - c_x h_y \tag{11}$$

$$p_y(h_x, h_y) = a_y - b_y h_y - c_y h_x \tag{12}$$

where a_i is the constant term, b_i is the sensitivity to own harvest and c_i is the cross-price sensitivity, then we can make the following statement:

Proposition 3

In the case of linear demand, the steady-state is determined exclusively by the sum of the cross-price parameters.

Proof: It is seen from the analysis above and eqs. (7) and (8) that the cross-price parameters only affect the steady state through the terms R_3 and R_4. In the linear case these terms can be written

$$\begin{aligned} R_3 &= a_x - 2b_x h_x - (c_x + c_y)h_y \\ R_4 &= a_y - 2b_y h_y - (c_x + c_y)h_x. \end{aligned}$$

Thus it is seen that the cross-price parameters enter the equations that determine the steady state in the form of the sum of the two parameters.■

In other words, no matter how asymmetric the economic and biological submodels are with respect to demand, cost structure and surplus growth function, if the cross-price parameters change value such that their sum remains the same, the steady state will remain unchanged. In practice this means that the two fish stocks can be quite different regarding economic, biological and technological aspects, if we let the cross-price parameters change values such that for example $c_x = 3$ and $c_y = 7$ instead of the other way around, it will not change the steady state.

4 DYNAMIC ANALYSIS

Not only the steady state, but also the optimal paths leading to the steady state are of interest, and, in particular, how they are affected by the cross-price parameters. As in the previous section, the case with stock-independent net revenue, in practice stock-independent costs, will be analyzed first. Thereafter the case where net revenue depends on the stocks, is investigated.

4.1 State-independent net revenue

Here we let net revenue depend on harvest only and not on the stock size. This is representative of fisheries with purse seine technology targeting schooling fish, and only the extreme case is investigated, that is no trace of the stocks in the net revenue function whatsoever, which implies that, in addition to $R_{32} = R_{41} = 0$, from earlier, we also have

$$R_1 = R_2 = R_{31} = R_{42} = 0$$

just like in Section 3.1. The first-order conditions corresponding to (5) and (6) then simplifies to:

$$\delta = f^{'}(x) + \frac{R_{33}}{R_3}\frac{dh_x}{dt} + \frac{R_{34}}{R_3}\frac{dh_y}{dt}$$

$$\delta = g^{'}(y) + \frac{R_{43}}{R_4}\frac{dh_x}{dt} + \frac{R_{44}}{R_4}\frac{dh_y}{dt}$$

This system can be solved for the time derivatives of the control variables yielding

$$dh_x/dt = \frac{R_4 R_{34}(g^{'} - \delta) - R_3 R_{44}(f^{'} - \delta)}{C} \tag{13}$$

$$dh_y/dt = \frac{R_4 R_{33}(g^{'} - \delta) - R_3 R_{34}(f^{'} - \delta)}{C} \tag{14}$$

and C denotes the determinant as earlier, assumed to be positive. It is immediately seen that in the case with stock independent net revenue, although the steady states are unaffected by the cross-price parameters, the optimal paths are affected.

Together with the dynamic constraints, (3) and (4), the equations (13) and (14) constitute a system of four non-linear first-order differential equations. In principle, this is a solvable system yielding the optimal time paths for $h_x(t)$, $h_y(t)$, $x(t)$ and $y(t)$. Due to the non-linearities, meaningful closed-form solutions are beyond expectation. The only approach, therefore, is to solve the system numerically.

In order to perform numerical analysis, special functional forms must be determined. Here linear inverse demand functions will be applied where the price of each species depends on both own harvest and the harvest of the other species as specified by eqs. (11) and (12). In addition it is assumed that the growth functions, f and g, are standard logistic surplus growth functions:

$$f(x) = r_x x \left(1 - \frac{x}{K_x}\right)$$

$$g(y) = r_y y \left(1 - \frac{y}{K_y}\right)$$

where r_i and K_i have the conventional interpretations as intrinsic growth rate and carrying capacity for $i = (x, y)$, see Clark (2010). The numerical specification of the above equations is given in appendix. The numbers are not meant to represent any real fisheries, rather they are meant to describe completely hypothetical, but still possible, fisheries with meaningful characteristics; in other words fisheries that very well might have existed.

First, the optimal steady state is found:

x	60
y	332.5
h_x	9
h_y	74.8125

From Proposition 1 we know that in this case the steady state is independent of the cross-price parameters c_i. In a non-linear four-dimensional system, there are multiple solutions, but fortunately, for the cases considered hereonly one of the solutions exist of positive real numbers in the feasible region which is

$x \in [0, K_x]$, $y \in [0, K_y]$ and $h_i \in \left[0, \frac{r_i K_i}{4}\right]$. Remember that with the logistic model $\frac{rK}{4}$ represents maximum sustainable yield.

In order to investigate the effect of cross-price parameters on the optimal time paths, we start by comparing the situation where both cross-price parameters are zero with the case where one of them is non-zero, namely $c_x = 0.08$ (see appendix). The first case (both parameters zero) represents two completely independent species, both biologically and economically, and the time paths for $x(t)$, $y(t)$, $h_x(t)$ and $h_y(t)$ are illustrated in Figures 1 and 2.[3]

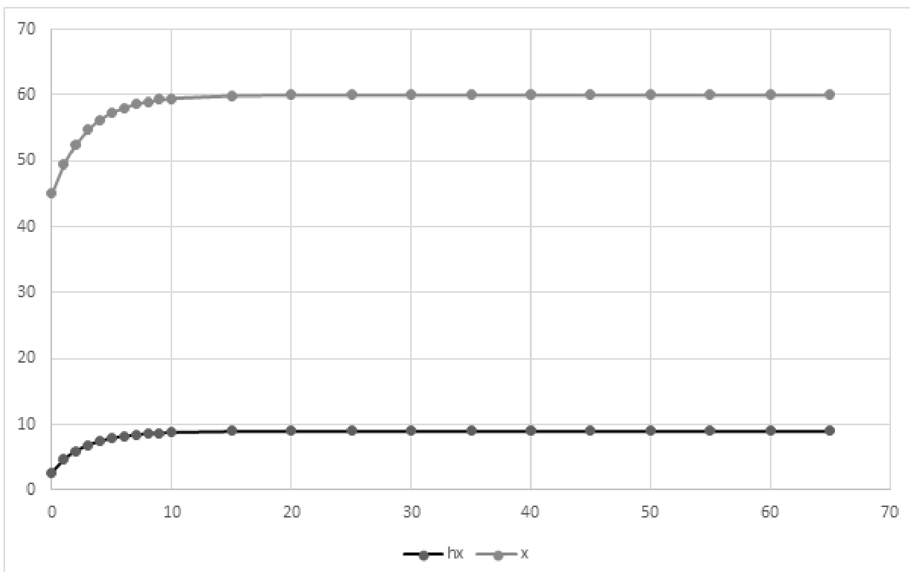

Figure 1. Stock and harvest development for species x when net revenue is independent of the stocks and there are no cross-price effects.

[3] The numerical solutions have been found using dsolve (numeric) in Maple 18.

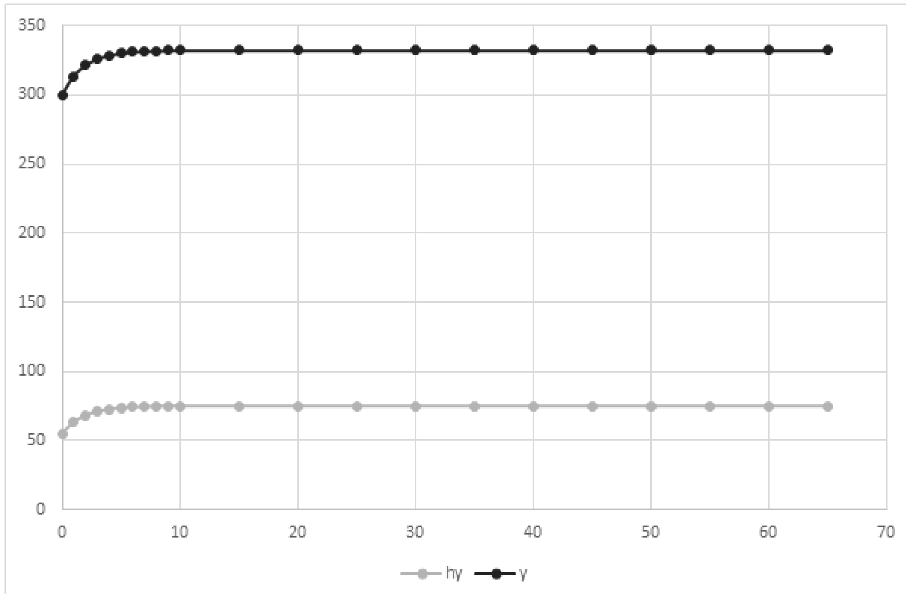

Figure 2. Stock and harvest development for species y when net revenue is independent of the stocks and there are no cross-price effects.

The stocks are assumed to be overexploited initially (like so many fish stocks around the world), and it is seen that the approach to the steady state is asymptotic due to the non-linearity (as opposed to the bang-bang approach resulting from linear models, see Clark (2010)). The boundary conditions applied here are $x_0 = 45$, $y_0 = 300$ and h_x and h_y at $t = 65$ equal to the optimal steady state harvest. It is reassuring to see that with these initial conditions the stock levels also approach the independently calculated steady states even when they are not restricted to it. I take this as a confirmation that the paths are really optimal.

Then we compare this with the case where we have a cross-price effect, namely $c_x = 0.08$, and with the same boundary conditions. This is illustrated in Figures 3 and 4.

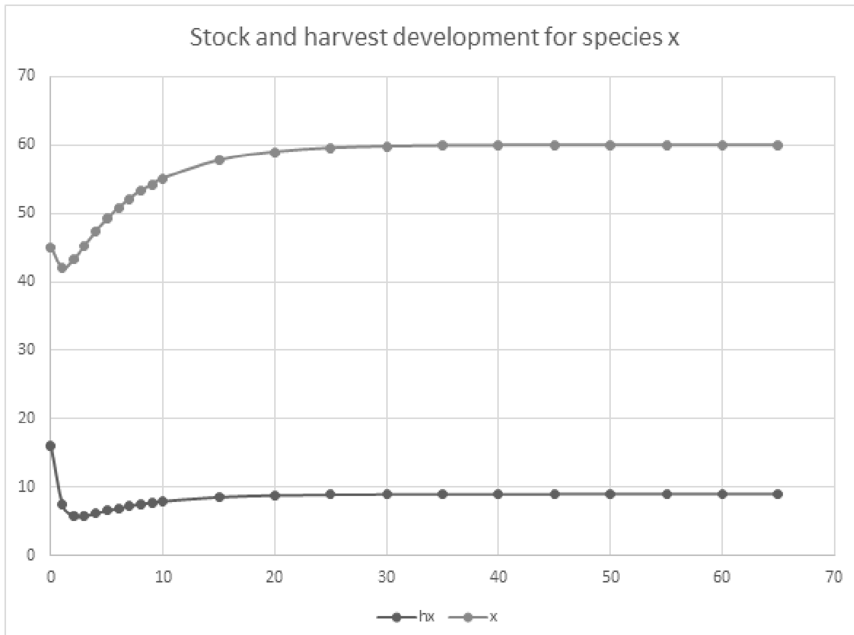

Figure 3. Stock and harvest development for species x when net revenue is independent of the stocks but the harvest of y affects the price of x.

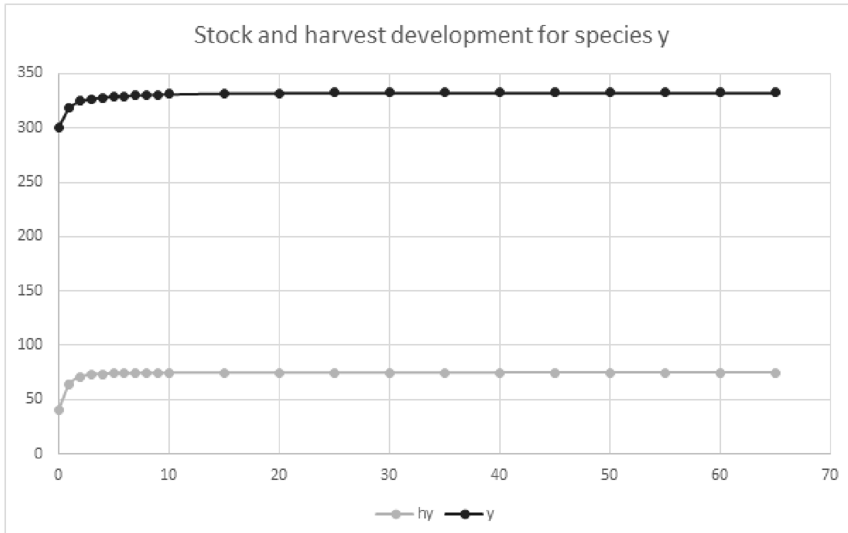

Figure 4. Stock and harvest development for species y when net revenue is independent of the stocks but the harvest of y affects the price of x.

The most noticeable features of these figures are that for species x, whose price is affected by the other species, there is undershooting in the stock path and both over and undershooting in the harvest path as seen from Figure 3. In the stock path, it is seen that the stock is first driven further down before it gradually starts moving up to the long-term steady state. This is a result of the harvest path where the harvest first is higher than the steady state level, then goes below the steady state level and then gradually approaches it. This is both an interesting, and very robust result. Figure 4 may look quite similar to Figure 2, but they are not identical. Harvest is initially lower but increases faster after cross-price effects are introduced. It is perhaps even more interesting to see how this affects the stock development. The stock of y increases a bit faster in the beginning with cross-price effect, but then settles on the same path towards steady state. This is the opposite of the x-stock development which even goes down initially. In other words, the behavior of the x-stock is less bang-bang like and the y-stock more bang-bang like with the cross-price effect from y on x.

4.2 State-dependent net revenue

In the section "Steady state analysis" there was significant difference between the cases with and without stock-dependent net revenue, in practice costs. It may therefore be interesting to investigate whether there is any noticeable difference in the dynamics case also. In this section the standard cost function derived from the Schaefer production is applied:

$$\kappa_x(h_x, x) = \frac{C_x h_x}{x}$$

$$\kappa_y(h_y, y) = \frac{C_y h_y}{y}$$

where the values for the parameters C_x and C_y are given in appendix.

First, the long-term optimum is calculated, and this is affected by the cross-price parameters as shown earlier. The steady state for the case without cross-price effects and for some combinations of parameter values are reported in Table 1.

Table 1. Steady states for some combinations of parameter values

	$c_x = 0$	$c_x = 0.08$	$c_x = 0$	$c_x = 0.08$
	$c_y = 0$	$c_y = 0$	$c_y = 0.01$	$c_y = 0.01$
x	77.1	110.6	78.7	121.1
y	409.6	413.4	410.2	413.0
h_x	9.4	7.3	9.4	5.8
h_y	75.5	75.4	75.5	75.4

According to Proposition 2, x will increase when c_y increases and vice versa, everything else equal. This is confirmed by the table. Typically the stock will also increase when the own price-parameter increases, but not necessarily so, as seen when c_y increases from 0 to 0.01 for $c_x = 0.08$. In this case the stock y decreases slightly.

Regarding the paths, Figures 5 and 6 illustrate the time-paths when there is no dependency between the species, just like in Figures 1 and 2.

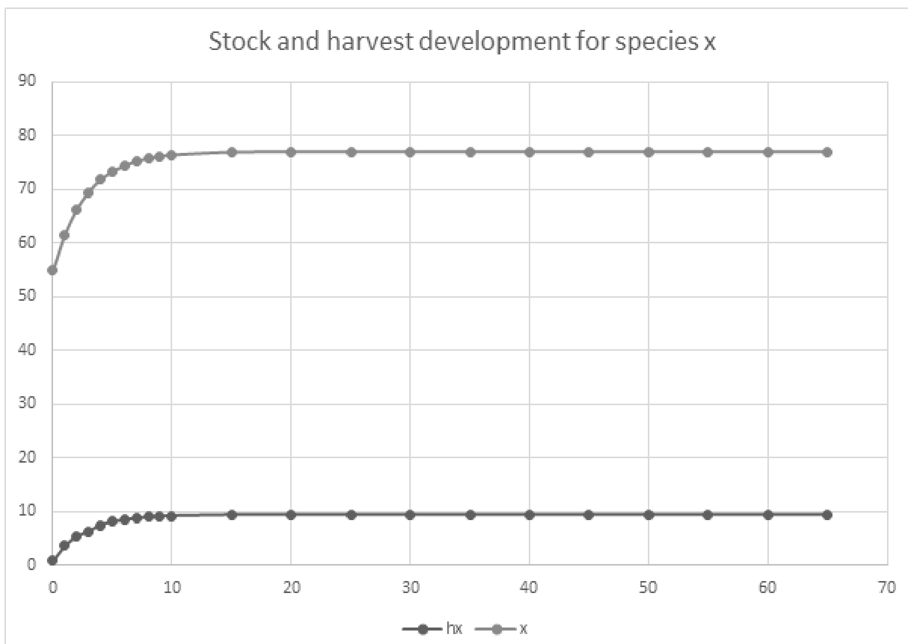

Figure 5. Stock and harvest development for species x when net revenue is stock-dependent and there are no cross-price effects.

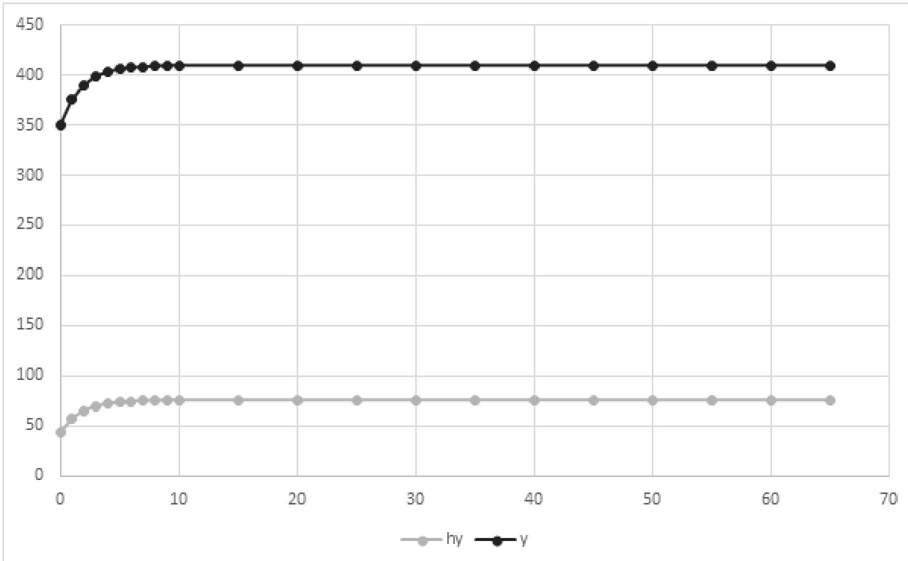

Figure 6. Stock and harvest development for species y when net revenue is stock-dependent and there are no cross-price effects.

It is seen that the paths increase monotonically and approach the steady state asymptotically without any sign of over- or undershooting, just as expected. Then it is interesting to compare this with the case where the cross-price parameter is positive, $c_x = 0.08$. This is illustrated in Figures 7 and 8, and again it is seen that the introduction of market interaction between the species leads to over- and undershooting for the harvest-development of the species whose price is affected by the other species.

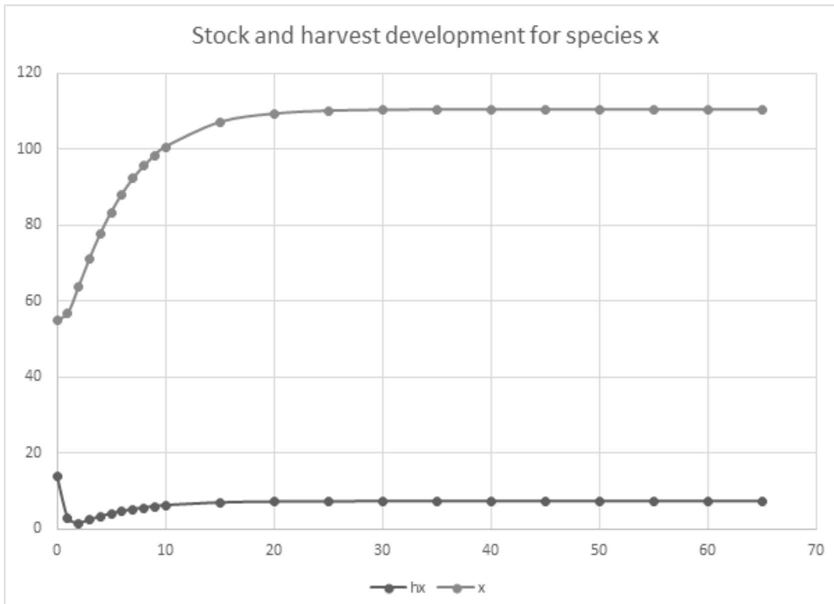

Figure 7. Stock and harvest development for species x when net revenue is stock-dependent, and the harvest of y affects the price of x.

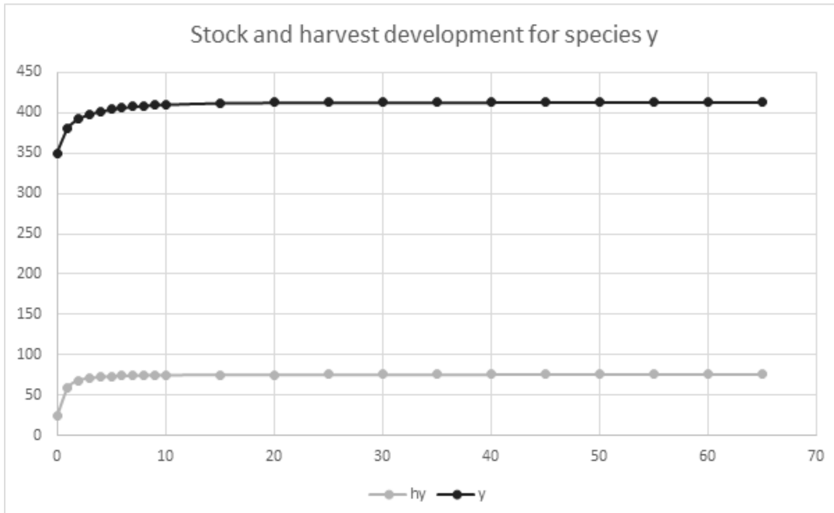

Figure 8. Stock and harvest development for species y when net revenue is stock-dependent, and the harvest of y affects the price of x.

Figure 6 and Figure 8, illustrating harvest and stock development for y with and without cross-price effects, may look very similar, but both the paths and the steady state are affected by the introduction of cross-price effects.

Thus, it is seen that whether net revenue is stock-dependent or not does not have any significant impact on the shape of the optimal time paths although it has significant impact on the steady states, as seen in previous sections. The shape of the time-paths are mainly affected by the cross-price parameters in this setting.

5 SUMMARY AND CONCLUSIONS

This article is about a two-species bioeconomic model where the only interaction between the species is in the market. In other words, there is no technical or biological interaction between the species. This may be relevant for a social planner, for example the managing authorities in a country, who has to deal with several species around the coast. These species may be located in different geographical areas and therefore do not interact biologically, but their products are sold in the same market.

It has been shown that whether cross-price elasticities have impact on the steady state or not, depends on the technology in the respective fisheries. In fisheries where effort and costs are independent of the total stock size, cross-price elasticities have no such effect. This is typically relevant for fish species with schooling behavior, and therefore harvested using purse seine technology. For demersal species, which typically are caught using bottom trawl, the cross-price elasticities actually affect the optimal size of standing stocks and corresponding harvest. More precisely, the qualitative effect is such that the presence of cross-price elasticities have a conservative effect on the stocks. In other words, the presence of a substitute in the market plays the same role as an additional cost. This was shown analytically in the section Steady State Analysis. This is a generalization of the same result from single-species models. A novel result found here is that, in the case of linear demand functions, it is the sum and only the sum of the cross-price parameters that affect the steady states, and not their composition or individual values.

In the section Dynamic Analysis it was shown that the optimal paths towards steady state are affected by cross-price elasticities, irrespective of technology. The effect is such that when the cross-price influence is sufficiently strong, the stock and harvest paths go from being monotonically increasing or decreasing to exhibit over- or undershooting. Overshooting is defined as region where the variable in question increases although it is already above the target level before it eventually approaches the target, and undershooting is defined as region where it decreases although it already under the target. In other words, regions on the time path where the variables move away from the target for a while before they come "back on the track". No trace of over- or undershooting have been found when the cross-price effects are removed.

The results presented here are fairly novel, and therefore there is scope for quite a bit of future research. This may include the combination of biological and market interaction, the combination of technological interaction and market interaction. And it may, of course, include other numerical examples, numerical analysis of other functional forms, and not least empirical investigation of particular cases.

6 APPENDIX

In this appendix the numerical specification applied in the analysis is summarized in the following table

r_x	K_x	r_y	K_y	δ
0.25	150	0.4	760	0.05
a_x	b_x	c_x		
10	0.1	0.08		
a_y	b_y	c_y		
15	0.02	0.01		
C_x	C_y			
200	1500			

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A two-species bioeconomic model is analyzed, but in contrast to most similar models, there is no biological interaction between the species, only economic. The interaction takes place in the market where the quantity of either species may affect the price of the other. The effects of cross-price elasticities on the optimal steady state and on the optimal paths in the sole-owner case are investigated both analytically and numerically. First, it is shown that whether cross-price elasticities have impact on the steady state or not, depends heavily on the technology in the fishery (e.g. purse seine versus trawl). Further, in the case of linear demand functions, the steady state outcome depends solely on the sum of the cross-price parameters and not their individual values. This is shown analytically. Secondly, in the investigation of optimal paths, numeric methods must be resorted to. It is shown that cross-price elasticities have interesting effects on the paths. More precisely, when cross-price elasticities are present and are sufficiently high, the paths go from being monotonic to feature over- or undershooting.

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